COMP 5660/6660 - Evolutionary Computing - Class Slides

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Computational Problem Solving

- Step 1: build abstract/computational model of the real-world

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- "Everything Should Be Made as Simple as Possible, But Not Simpler"[1]
- Step 3: map solution back to real-world

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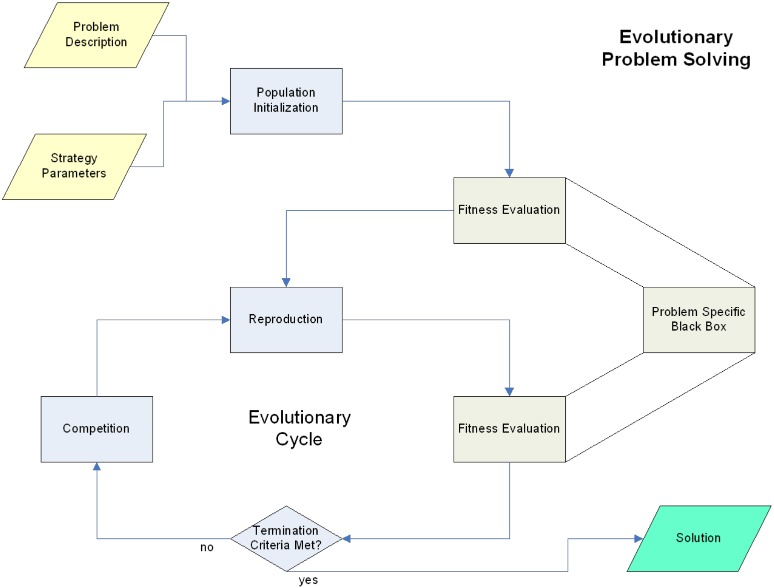
- **Evolutionary Algorithms (EAs)** can be described as a class of *stochastic*, *population-based* BBSAs inspired by *Evolution Theory*, *Genetics*, and *Population Dynamics*

Evolutionary Cycle



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- Alleles - variant forms of a gene

Nature versus digital realm

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Population	Spatial embedding and dynamic size	Typically unstructured and constant size

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- F does not need to be injective, but realize there is less to search if F is injective so there should be sufficient compensation, such as limiting F(G) to valid solutions in a constraint satisfaction problem.

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- Recombination: stochastic multi-ary variation operator

Canonical Bit-string Variation Operators

- Mutation
  - Bit-flip

Canonical Bit-string Variation Operators

- Mutation
  - Bit-flip
  - $E[\#flips] = L \cdot p_m$
- Crossover
  - Choose a random point on the two parents
  - Split parents at this crossover point
  - Create children by exchanging tails
  - P_c typically in range 60-90%, but 100% is quite typical too

Integer mutation

- Random Reset (cardinal attributes)

Integer mutation

- Random Reset (cardinal attributes)
- Creep Mutation (ordinal attributes)

Floating-point mutation

- Uniform

Floating-point mutation

- Uniform
- Non-uniform (e.g., Gaussian, Cauche, Levy)