# COMP 5660/6660 - Evolutionary Computing - Lecture Slides

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# Computational Problem Solving

- Step 1: build abstract/computational model of the real-world

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- Step 2: solve computationally in abstract model
- "Everything Should Be Made as Simple as Possible, But Not Simpler"[1]
- Step 3: map solution back to real-world

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# Computational Problem Classes

- Decision problems
- Optimization problems
- Modeling (aka system identification) problems
- Simulation problems
- Search problems

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- Local versus global optima
- Unimodal versus multimodal problems

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- A **Black-Box Search Algorithm (BBSA)** is a meta-heuristic which iteratively generates trial solutions employing solely the information gained from previous trial solutions, but no explicit problem knowledge
- **Evolutionary Algorithms (EAs)** can be described as a class of *stochastic*, *population-based* BBSAs inspired by *Evolution Theory*, *Genetics*, and *Population Dynamics*

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- Solution availability during computation
- Robustness
- Inherent parallelism

# EA Cons

- Fitness function and genetic operators often not obvious
- Premature convergence
- Computationally intensive
- Difficult parameter optimization

# Biological Metaphors - Darwinian Evolution

- Macroscopic view of evolution
- Natural selection
- Survival of the fittest
- Random variation
- Genetic drift

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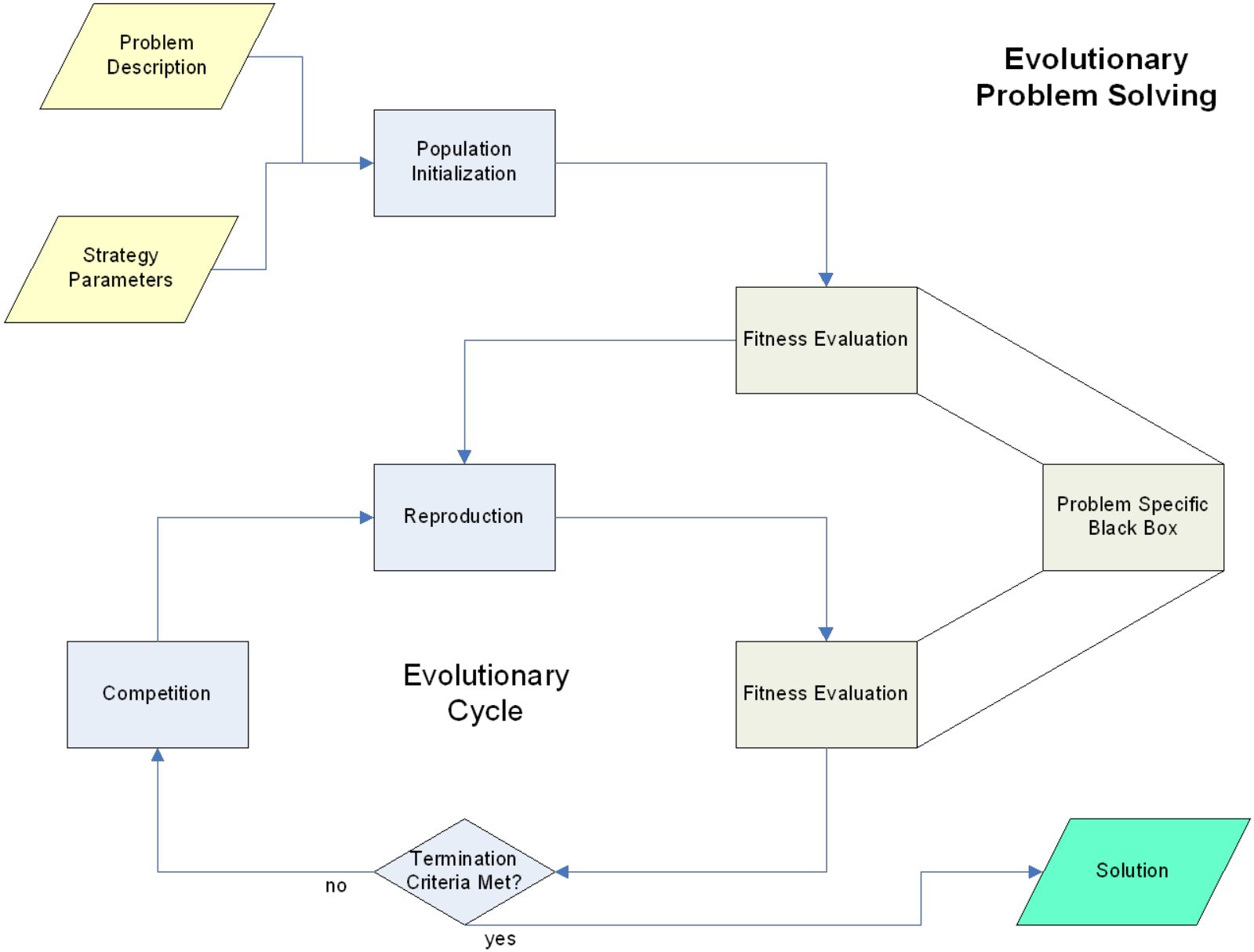
- Alleles - variant forms of a gene

# Evolutionary Cycle



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- If $F$ is not surjective and $x^* \notin F(G)$, then the EA cannot find the global optimum. Therefore one should think twice before choosing a non-surjective decoder function if one cannot guarantee that the global optimum is still reachable.
- $F$ does not need to be injective, but realize there is less to search if $F$ is injective so there should be sufficient compensation, such as limiting F(G) to valid solutions in a constraint satisfaction problem.

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2. Modify $fitness(p)$ to exclude items that would exceed $C_{max}$

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